

A comparative analysis on efficiency using slacks-based DEA, standard DEA versus network DEA

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Abstract

The health care sector is a complex system which involves not only the health provider organizations but also the surrounding environment. In specific, hospitals are influenced by external (national health and financial environment) and internal (staff productivity and structure fluxes) variables. Therefore, new forms to evaluate hospital's efficiency, especially the ones which incorporate services' interconnections, enhance the health sector value. Hence, the development of network DEA methodology capable of being applied in any hospital is the first step to uncover more sophisticated techniques. Accordingly, this study proposes a comparison between the most used methodology for performance evaluation, DEA, and the most recent improvement, network DEA. By taking into consideration the current growth of private hospitals, the efficiency analysis was applied to the largest private healthcare provider in Portugal, CUF. Therefore, the present dissertation is filling two knowledge gaps, lack of academic efficient analysis in the private sector and the absence of comparative studies between standard DEA and network DEA. The obtained results for the new model revealed an increase in the number of efficient hospital's services units, as well as a general decrease in the number of global efficient hospital units. Finally, by crossing information with the business viewpoint, the inclusion of hospital fluxes is enlightened. The network DEA provides a holistic view of hospital dynamics.

Keywords: Efficiency; Methodologies' Comparison; Network DEA; Private sector;

1. Introduction

Efficiency, independently from the sector it is applied, is an important factor in the regulation of the resources used and produced (O'Brien et al., 2012; Kao, 2017). Being on a period when waste is economically, environmentally, and socially undesired, evaluating efficiency is crucial (Ekins and Hughes, 2016).

In specific for the health sector, where we are dealing with a fundamental human right, as envisaged by WHO constitution (1946), "... *the highest attainable standard of health as a fundamental right of every human being.*", it is essential to improve the quality of delivered care. Surprisingly, several studies reported that, population health is directly correlated to the hospital's efficiency (Chisholm and Evans, 2010; Mossialos and Grand, 2019). Therefore, inefficiency in the health system can result in the deny of treatment and health improvement to patients, due to the excessive consumption of resources, who would otherwise have received treatment if resources had been better managed (Cylus et al., 2016).

Nevertheless, the elaboration of a strategy

based on complete measurements have not been properly executed, as supported by an WHO study. In particular, for Portugal, where this dissertation is being developed, Healthy, prosperous lives for all: the European Health Equity Status Report stated that Portugal is one of only four countries (of 33 analysed) that reduced public health expenditure between 2000 and 2017.¹ Over 400 published applications have used these methods in the past (Hollingsworth, 2008; Hollingsworth and Scott, 2012). As this falling investment is preventing the modernisation of hospitals and replacement of obsolete medical equipment, private care is expanding (The Lancet., 2019). Overall, in 2014, about 10 847 deaths were deemed to be avoidable through the delivery of higher quality and more timely health. As a consequence of these factors, private VHI has been growing over the years, as much as has been noticed in general in EU, since the percentage is equivalent, being at 5%

¹ WHO. (2019). Healthy, prosperous lives for all: the European health equity status report. Technical report. Retrieved from <http://www.euro.who.int/en/publications/abstracts/health-equity-status-report-2019> [Accessed: February 28, 2020]

of the health financing (OECD, 2017). Therefore, the study was conducted using the data set of the largest private healthcare provider in Portugal, CUF.

In regards to efficiency analysis in health sector, in the last three decades has been mainly performed using “black-box” models, such as DEA and SFA. It means that the hospital is considered as a whole, with global input and output variables, ignoring internal interactions (Hollingsworth, 2016).¹ Specifically, DEA (Charnes et al., 1978, 1994) has been applied in more than 90% of health care setting, since it can account for multiple inputs and outputs, varying weights and returns to scale (Hollingsworth, 2016; Kao, 2017).

Nevertheless, there are several services (divisions) in one single hospital and inefficiency problems can then arise from relations that are not being considered. Therefore, the source identification is potentially skewed, resulting in inappropriate correction, and possible waste of even more resources. Consequently, it is essential to assess these service interactions to evaluate hospital performance. By applying network DEA, (Färe and Grosskopf, 2000; Kao, 2017) room is made to uncover the potential of innovative and more specific efficiency methodologies. It may facilitate the setting of benchmarks on funding specific services or entire hospitals. For that reason, by comparing an innovative methodologies, network DEA, with the most used one, standard DEA, this thesis will untangle the potential of the newest methodology.

This extend abstract comprises six sections. Section 1 is the introduction that describes the motivation for using non-“black-box” DEA, in particular in the CUF case study. Section 2 encloses the context and problem on which this work is based on. Section 3 visits the literature and provides resumed review on published papers. Section 4 presents the proposed methodologies to be compared. Section 5 contains the outline of the case study, the results and their discussion. There is also an important subsection in section 5 where the business viewpoint is explored. The conclusions, recommendations and limitations are disclosed in the final Section 6.

Hopefully this will result in a methodology’s improvement for efficiency analysis in the health sector worldwide. Specifically in Portugal, which has been ignored due to its lack of long-term strategy and bad resource allocation. Therefore, our sincere wish is that this study contributes to a better Portuguese future, in particular for the health sector.

2. Context and Problem

The combination of a sector which incorporates the universal human right to health and an era of not

only cost reduction but also use maximization of available resources, there is an opportunity space to research new methods in this field. Not forgetting the particularity that resources are limited and there are unlimited needs.

These sustainability problems, not only in the health sector but also in other areas, resulted in the United Nations Members States setting 17 SDGs, in 2015, to be met by 2030. For instance, it includes a broad health goal, “*Ensure health lives and promote well-being for all at all ages*”.² However, enormous gaps remain between what is achievable in human health and where global health stands today, and progress has been both incomplete and unevenly distributed. Although, the first impression is that lack of quality services is only related to poor countries, high-income countries have fallen short on the quality scale as well. The lack of coordination within and among healthcare providers and networks appears to be the greatest concern. Overall, it can be seen as another opportunity space to improve the resources management, reducing costs without compromising quality. In other words, guaranteeing the sustainability of the health system and quality care of its patients (National Academies of Sciences et al., 2018).

2.1. Private Sector

The dissertation’s data was provided by one of the main Portuguese’ private hospital organizations. Therefore, it is essential to understand the private sector context.

Private hospitals are owned and operated by an organization other than the state, and they can be categorized as for-profit and non-profit companies. In particular, the Portuguese health market is not innovative in the demand for private hospitals’ growth, see Figure 1.³ It is the reality in several other countries, such as the USA, United Kingdom, Germany, and Austria, where the advantages of private providers are perceived (Kruse et al., 2018). Faster access to treatment, the opportunity to choose the healthcare provider, and the comfort of the surroundings are some of the main advantages felt by the patients (Sagan and Thomson, 2016).

Thus, Governments has began to perceive the private sector as a possible solution to challenges,

² WHO (2019). World health statistics 2019: monitoring health for the sdgs, sustainable development goals. Retrieved from <https://www.un.org/en/sections/issues-depth/health/> [Accessed: March 26, 2020]

³WHO: European Health Information Gateway. “For-profit privately owned hospital, total number”.[Online]. Available: https://gateway.euro.who.int/en/indicators/hlthres_56-for-profit-privately-owned-hospitals-total/visualizations/#id=27851 [Accessed March 28, 2020]

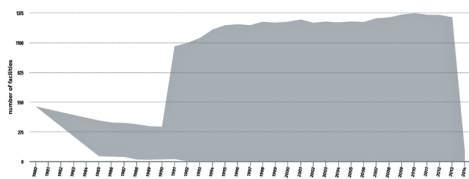


Figure 1: Total number of for-profit privately owned hospital. Source: WHO - European Health Information Gateway.

such as the coverage by public health service.⁴ Although there are several noticed advantages in the private sector, there is one factor which must be recognized first, the lack of published information in this area.⁵ Due, in part, to the private sector's failure in publishing data where the performance is evaluated. These challenges should be addressed with brevity since the private sector has been developing recently and serves more than half of the population in some countries (Basu et al., 2012).

In regarding CUF's data acquisition, it is an essential first stage in order to perform efficient analysis. The electronic records are not a recent subject when it comes to the CUF routine, therefore the activity and billing analysis have been facilitated. Nevertheless, the increasing number of clinics and hospitals, as well as the number of informatic applications have brought challenges. The analysis made, in historical terms, for control and efficiency issues requires the use of differentiated human resources' knowledge and the Microsoft Excel calculation tool. In regards of efficiency language, the organization analysis is based on ratio analysis, a dependent methodology. It relies on the number of performance dimensions. Although an investment path has been made in CUF, especially regarding the use of business intelligence tools, what is proposed in this thesis is an innovative approach that correlates the different hospital's dimensions. (Hollingsworth, 2016; Kao, 2017).

3. Literature Review

Several methodologies, parametric and non-parametric can be used to evaluate hospital performance. From the literature review and in agreement with Ozcan (2008), viz.: (1) Ratio analysis; (2) Least-squares regression (LSR); (3) SFA; (4)

⁴Clarke, D., Doerr, S., Hunter, M., Schmets, G., Soucat, A., and Paviza, A. (2019). The private sector, universal health coverage and primary health care. technical series at the global conference on primary health care. Retrieved from <https://www.who.int/publications-detail/the-private-sector-universal-health-coverage-and-primary-health-care>. [Accessed: March 30, 2020]

⁵Balabanova, D., Oliveira-Cruz, V., and Hanson, K. (2008). Health sector governance and implications for the private sector. Retrieved from <https://www.r4d.org/resources/health-sector-governance-implications-private-sector/>. [Accessed: April 31, 2020]

DEA.

Ratio analysis is mainly used for productivity in healthcare financial management. Ratios are the relationship and comparison between one input and one output (Martins, 2004). However, it depends on the number of performance dimensions, which have to be aggregated into compatible units or within one unit over different time periods. To overcome this disadvantage, LSR allow the use of multiple inputs and outputs. The benefits of LSR are perceived when technical change of a time-series data is required and when one is investigating scale economies. Therefore, it became one of the most popular parametric methods in efficiency evaluation. However, the LSR does not identify specific inefficient units nor the best performances, and it requires a pre-specified production function due to its parametric formulation (Ozcan, 2008). As for SFA technique, it is normally used to estimate production and cost functions. While explicitly accounting for the existence of inefficiency. When comparing to other parametric method, SFA assumes that deviations from the efficient frontier are due to a noise factor. Consequently, SFA is ideal for hypotheses' testing and to measure not only technical and allocative efficiencies, but also profit and cost analysis. Nevertheless, results are highly sensitive to the estimation decisions made. The challenges associated with model construction, such as specification of functional form and identification and extraction of efficiency estimates promoted the developed of other methods, such as DEA (Jacobs et al., 2006). This technique, used in the first part of this thesis, assumes that not all units are efficient and allows the use of multiple inputs and outputs in a linear programming model. Afterwards it computes a single efficiency score per observation.

Nonetheless, DEA also presents limitations. For instance, since it is deterministic it does not incorporate noise as SFA and are therefore more susceptible to the outliers (Hollingsworth, 2003; Ruggiero, 2007). Although there will never be the perfect methodology, it is possible to overcome some challenges. For instance, it was identified that none of these techniques recognizes internal connections. For that reason the present dissertation proposes an almost unexplored methodology in healthcare sector. In contrast to conventional DEA, where a system is considered as a "black-box", network DEA acknowledges its internal assembly to generate more enlightening results. For example, common DEA ignores: (1) that a system may be globally efficient, even if all its subsystems are not (Kao and Hwang, 2008), and (2) the subsystems of a DMU may have worse performances than the ones of another DMU, with the former be-

ing more efficient than the final (Kao and Hwang, 2010). Although various structures can be used to perform network systems, it has not been largely applied in health sector. Charnes et al. (1986), the first to explore this methodology, applied a basic two-stage system. In this study, he observed the effect of army recruitment in a two-process operation, allowing to detail large exercises into smaller ones. The process facilitated the identification of real impact of input factors. Nonetheless, it was not until 2000 that Färe and Grosskopf introduced the term 'network DEA'. Therefore, it was crucial to understand the extension of the application of this technique. In 30 studies analysed, only 3 used network DEA; and none of them compared standard with network DEA. For that reason, it is proposed to seal this knowledge gap.

To finalize, it was also analysed the use of different techniques for public and private sector. No differences have been noticed. The most common methods are, for both sectors, DEA and SFA when it comes to measuring health efficiency (Hollingsworth, 2008; Hollingsworth and Scott, 2012; Hollingsworth, 2016; Jaafaripooyan et al., 2017). As confirmed by several papers which have compared the efficiency of private (for-profit and non-profit) and public hospitals. Therefore using the same methodology for both sectors (Marinho, 2001; Tiemann and Schreyögg, 2009, 2012; Hsiao et al., 2018).

4. Methodologies

4.1. DEA

Although there are several methodologies which can be used to determine the DMUs that form the efficient frontier, vide supra, DEA is able to evaluate DMUs using multiple inputs to produce multiple outputs. However, radial DEA measures, which can be performed from either the input or output side, have difficulty in defining weakly efficient DMUs. Therefore, these units cannot be compared with inefficient DMUs. Additionally, input and output factors must be considered separately, resulting in inconsistent efficiency scores (Kao, 2017). Slacks-based models emerge to overcome this weakness of other DEA models, culminating in the chosen methodology for the present dissertation. For that reason, and since the healthcare sector is pointed out as one of the most complex, this work relied, on the first part, on a slacks-based DEA to measure efficiency. The chosen model, considering VRS, was initially proposed by Charnes et al. (1985), Equation 1. The application of VRS is justified by the imperfect competition, constraints on finance, external influences, and regulatory constraints found in healthcare sector, which often results in organizations operating at an inefficient scale. This way, it is possible to include all DMUs, even if not operating at an optimal scale.

$$\max \sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \quad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^n \lambda_j X_{ij} + s_i^- = X_{ik}, \quad i = 1, \dots, m; \quad (2)$$

$$\sum_{j=1}^n \lambda_j Y_{rj} - s_r^+ = Y_{rk}, \quad r = 1, \dots, s; \quad (3)$$

$$\sum_{j=1}^n \lambda_j = 1, \quad j = 1, \dots, n; \quad (4)$$

$$\lambda_j, s_i^-, s_r^+ \geq 0, \quad \left\{ \begin{array}{l} j = 1, \dots, n \\ r = 1, \dots, s \\ i = 1, \dots, m \end{array} \right\} \quad (5)$$

The process is repeated n-times, once for each DMU_j , with $j = 1, \dots, n$ since the model is searching for an efficient point in the production frontier that is the most distant from the DMU under analysis. DMU_k 's efficiency is computed using the geometric mean, according to Equation 6, since it is the best fit for compounding numbers expressed in different units and introducing a certain degree of non-compensability between indicators.

$$E_k = \frac{\left(\prod_{i=1}^m 1 - \frac{s_i^-}{x_{ik}} \right)^{\frac{1}{m}}}{\left(\prod_{r=1}^s 1 + \frac{s_r^+}{y_{rk}} \right)^{\frac{1}{s}}} \quad (6)$$

4.2. Network DEA

In the literature, measuring systems' efficiency with network DEA models has been achieved using various models. According to Kao (2014), there are nine types of models. However, they can be grouped into three (Kao, 2009), since all have a multiplier, an envelopment, and a slacks-based form. First, the independent model, assumes that each division is an independent DMU, measuring their respective efficiencies by applying conventional DEA models. For the second type, connected models emerge as an alternative to independent models. The last category, relational models, rests in the slacks-based form, combines the two previous concepts. The intersection of independent and connected models, allows relational models to measure system and its divisions' efficiency (Kao, 2013). Furthermore, since one of the main goals of this dissertation is to compare the system's performance using the DEA model versus the network DEA, the models must have similar methodologies. Therefore, the proposed model is slacks-based using a matrix-type structure, based on Pereira et al. (2020).

$$\max \sum_{w=1}^W s_w^{(p,p')+) + \sum_{r=1}^s s_r^{(p)+} + \sum_{i=1}^m s_i^{(p)-} + \sum_{t=1}^T s_t^{(p',p)-} \quad (7)$$

$$\text{s.t.} \quad \sum_{j=1}^n \lambda_j^{(p)} z_{wj}^{(p,p')} - s_w^{(p,p')+) = z_{wk}^{(p,p')}, \quad \left\{ \begin{array}{l} j = 1, \dots, n \\ p = 1, \dots, P \\ p' = 1, \dots, P' \\ w = 1, \dots, W \end{array} \right\}; \quad (8)$$

$$\sum_{j=1}^n \lambda_j^{(p)} y_{rj}^{(p)} - s_r^{(p)+} = y_{rk}^{(p)}, \quad \left\{ \begin{array}{l} j = 1, \dots, n \\ p = 1, \dots, P \\ p' = 1, \dots, P' \\ r = 1, \dots, s \end{array} \right\}; \quad (9)$$

$$\sum_{j=1}^n \lambda_j^{(p)} x_{ij}^{(p)} + s_i^{(p)-} = x_{ik}^{(p)}, \quad \begin{cases} j = 1, \dots, n \\ p = 1, \dots, P \\ p' = 1, \dots, P' \\ i = 1, \dots, m \end{cases}; \quad (10)$$

$$\sum_{j=1}^n \lambda_j^{(p)} z_{tj}^{(p',p)} + s_t^{(p',p)-} = z_{tk}^{(p',p)}, \quad \begin{cases} j = 1, \dots, n \\ p = 1, \dots, P \\ p' = 1, \dots, P' \\ t = 1, \dots, T \end{cases}; \quad (11)$$

$$\sum_{j=1}^n \lambda_j^{(p)} = 1, \quad j = 1, \dots, n; \quad (12)$$

$$\lambda_j^{(p)}, s_w^{(p,p')+}, s_r^{(p)+}, s_i^{(p)-}, s_t^{(p',p)-} \geq 0, \quad \begin{cases} j = 1, \dots, n \\ w = 1, \dots, W \\ r = 1, \dots, s \\ i = 1, \dots, m \\ t = 1, \dots, T \\ p = 1, \dots, P \\ p' = 1, \dots, P' \end{cases} \quad (13)$$

The optimal division for a generic DMU_k is also computed using the geometric mean, as expressed by Equation 14, while the system efficiency is measured according to Equation 6. Although the expression, for global efficiency, is the same for DEA and network DEA, the determination of the slacks, $s_i^{(p)-}$ and $s_r^{(p)+}$ depends on other constraints, which may result in different efficiency scores.

$$E_k = \frac{\left(\prod_{i=1}^m 1 - \frac{s_i^{(p)-}}{x_{ik}^{(p)}} \right)^{\frac{1}{m}} \left(\prod_{t=1}^T 1 - \frac{s_t^{(p',p)-}}{z_{tk}^{(p',p)}} \right)^{\frac{1}{T(p',p)}}}{\left(\prod_{w=1}^W 1 + \frac{s_w^{(p,p')+}}{z_{wk}^{(p,p')}} \right)^{\frac{1}{W(p,p')}} \left(\prod_{r=1}^s 1 + \frac{s_r^{(p)+}}{y_{rk}^{(p)}} \right)^{\frac{1}{s}}} \quad (14)$$

Yet, the choice of methodologies is not the only concern. The number of variables included in the model must also be taken into consideration, since DEA is sensitive to this parameter. The lack of discrimination between efficient and inefficient DMUs often arises when there is a relatively large number of performance variables when compared to the number of DMUs. In the literature, this is often referred to as the “curse of dimensionality” (Charles et al., 2019). Under this study, seven inputs were selected in a mixed typology to cover not only the labour contribution but also the financial one. Regarding the outputs, three were selected. Thus, by respecting the Equation 15, proposed by Charnes et al. (1985), it is possible to avoid this curse.

$$n \geq \max(m \times s, 3(m + s)) \quad (15)$$

Summing up, in the first part of this thesis, DEA is going to be used to evaluate hospitals and services efficiency. For the second part network DEA evaluates the hospitals’ global and partial efficiency. Lastly, by performing both methodologies for the same hospitals’ set it is possible to execute a plausible comparison in the third part.

5. Results & discussion

In the fifth chapter, the empirical application of the DEA and network DEA methodology is evaluated. It should be clear by now that the main goal of the case study engaged in this dissertation is to evaluate and compare the performance of the main hospitals of CUF organization, using standard DEA and network DEA. Therefore, firstly, the DMUs were defined; secondly, input and output variables were selected, as represented in Tables 1 and 2 ; and lastly, the efficiency analysis of the data collected from the CUF database.

5.1. Data and variables

The three biggest CUF hospitals - CUF Descobertas hospital (HDSC), CUF Infante Santo hospital (HCIS) and, CUF Porto hospital (HPRT) - regarding the number of clients and the number of beds, were selected as the subject of analysis. In order to enlarge the number of DMU, and prevent the appearance of the “curse of dimensionality” issue, the analysis is performed over three years (2017, 2018, and 2019) and each DMU corresponds to a different month, resulting in 108 DMUs.

In regards to the application of the standard DEA to each service, the variables used are described in Table 1. Nevertheless, since the objective is to compare the results with the ones posteriorly obtained applying network DEA, both variables set must be in agreement. Therefore, six inputs and two outputs for Division 1; seven inputs and one output for Division 2, six inputs and two outputs for Division 3.

As for the definition of network DEA’s variables, it is necessary not only to take into consideration variables that connect between divisions, as represented by Figure 2, but also the similarity with the variables used in the standard DEA. Therefore, Table 2, represents the selected variables, indicators, respective descriptions, and assigned divisions.

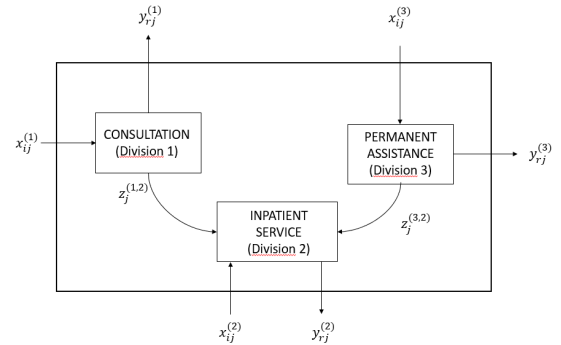


Figure 2: Schematic diagram of *Hospital_j* interactions with the environment and between divisions 1-3, Consultation, Inpatient Service, and Permanent Assistance.

5.2. Results and Discussion

Both slacks-based standard and network DEA were implemented in *Python*, using a *numpy* package to manage the numerical variables.

Standard DEA

The global efficiency when standard DEA was applied resulted in 63 efficient DMUs, which corresponds to 58.3% of total DMUs and the majority of global efficient DMUs are found in the CUF Porto hospital. Hereof partial efficiency, 48 DMUs were identified as efficient for Division 1 - Consultations; 46 DMUs for Division 2 - Hospitalization; and 21 DMUs for Division 3 - Permanent assistance. None of these divisions had a particular geographic or temporal distribution.

In comparison with the other divisions, Division 3 is the less efficient. This result is understandable since permanent assistance is being evaluated, which corresponds to the most dependent on external variables, like national health in that moment of the year.

Network DEA

In regard of global efficiency, only 20 DMUs globally efficient, which corresponds to 18.5%. Therefore, when the internal correlations between divisions are taken into consideration the efficiency decreases. Another observation, is the different in the identification of most efficient hospital, when in standard DEA it was HPRT in network DEA, it is HCIS. Nevertheless, not all of these DMUs are efficient in every service, and not every inefficient DMU is inefficient in every service. From the 20 globally efficient DMUs, eight are not efficient in every service under analysis, viz., HCIS_Abril2017, HCIS_Aug2017, HCIS_Nov2017, HDSC_May2017, HCIS_Jun2018, HCIS_Aug2018, and HDSC_May2018. Although, all of them are efficient in at least 2 of the 3 divisions under analysis. Despite the services of these DMUs have worse performance than the other ones, globally, hospitals are also seen as efficient, as previously observed by (Kao and Hwang, 2010). There are also no reported cases of DMUS that are globally inefficient, but efficient in every service.

Although the number of globally efficient DMUs is smaller for network DEA than for standard DEA, in partial efficiency, more DMUs are identified as efficient for network DEA. For instance, Division 1 presents 52 efficient DMUs, Division 2 shows 73 efficient DMUs, and in Division 3, 32 DMUs.

5.3. Methodologies' comparison

The following analysis is facilitated by observation of Tables 3 and 4.

Partial Efficiency

Deepen analysis results in observing that all DMUs identified as efficient when applying stan-

dard DEA were also efficient when network DEA was used. Another aspect that may be important to stress is that Division 2 is the only division that had a connection with more than one division, as observed in Figure 2. More specifically, three internal variables were entering Division 2 and it is the one which presents the biggest differences in the number of DMUs identified as efficient for both methodologies. Even more interesting, is that Division 1, the one with less internal variables is the one that has the lowest difference between standard and network DEA.

Global Efficiency

The first obvious observation is that the difference between standard DEA and network DEA is even bigger when the global analysis is being done. However, that is not the only interesting observation. For instance, contrary to the partial efficiency, for global efficiency, the standard DEA identifies more DMUs as efficient than network DEA. Another interesting aspect was noticed when a deeper analysis was performed. Not all the DMUs which are efficient for standard DEA are efficient for network DEA, and the other way around was also verified. Reflecting on the information provided, there are 51 DMUs that are only recognized as efficient when applying standard DEA and eight DMUs which are only identified as efficient when network DEA is used. From all the DMUs identified as efficient, only 12 DMUs are identified as efficient for both methodologies

5.4. Business viewpoint

The unbiased efficiency analysis, based on objective data, is crucial. Nonetheless, this analysis can never be dissociated from the business analysis. By including the healthcare business' point of view, the full scope of efficiency analysis will be covered.

CUF Infante Santo

The decrease of efficiency observed in 2019 is concordant with the reinforcement of medical staff. It does not only increase the FTEs but also the associated hospital's costs. The expansion of medical staff is a normal procedure to integrate human resources in the organization dynamic before the opening of new hospitals. Afterward, CUF Tejo, a new hospital, would inaugurate in 2020. Therefore, the incrementation of these two input variables, without the increase in the number of consultations results in the observed decrease in efficiency.

CUF Descobertas

The opening of building 2 in July of 2018, involved the upgrade of several medical specializations. Therefore, it was also verified an increase in FTEs which takes some time to be reflected in the increase of clients. These variations can justify the non-existed efficient DMUs in the second half

of 2018. However, there is not only an increase in the FTEs but also the costs associated with the equipment of the new building. Only more than one year later, in October of 2019, there is an efficient DMU identified in CUF Descobertas. Nonetheless, there is another historical moment, the legionella in February of 2018. CUF Descobertas also suffered an outbreak of legionella which certainly affected not only the number of clients and hospitalization but also the cost of disinfection and medical staff. These fluctuations are detected in the decrease of efficiency score in February of 2018.

CUF Porto

The CUF Porto which seems to be the least efficient when network DEA is applied but the most efficient for standard DEA has almost no historical facts to justify this inefficiency. There is only the accreditation of the International Joint Commission in June of 2018. This accreditation may have led to some entropy in the hospital's organization resulting in inefficiency. Nevertheless, this fact would not justify the non-existent efficient DMUs since October 2017.

General Factors

In 2018, the CUF organization, invested in the renovation of hospital equipment which influenced the total costs from thereon. This appears to be concordant with the decrease of efficiency throughout the years. In 2019, to add to this cost incrementation, there was also officialized the nursing career, resulting in retention policy and therefore an increase of nurses' FTEs. Although this has an incredible effect on the life quality of these professionals, it will also result in more inputs that are not immediately reflected in the increase of outputs, decreasing the immediate global efficiency.

6. Conclusions

Essentially, if, on one hand, these observations demonstrated the dependency of efficiency on the internal connections between division, as observed in the partial efficiency results, on the other hand, it is incomplete for the formulation of final conclusions regarding the differences between standard and network DEA for global efficiency. Nevertheless, this dissertation fills the knowledge gap in the literature respecting to the comparison of these two methodologies and provides healthcare manager and policymakers ability to distinguish methodology results and better support decision-making.

The efficient DMUs, although not exactly the same, are in general more demanding when applying the network DEA, pointing out only 18.2% in a total of 108 DMUs instead of the 58.3% pointed by standard DEA. For this reason, network DEA appears to be a more enlightening methodology,

since it includes internal system fluxes and consequently separates partial and global efficiency scores, providing a holistic view of the hospital dynamics.

6.1. Recommendations

Private healthcare sector's costs are mostly associated with medical staff (Bert, 2013). However, the inefficiency does not result from the increase of medical staff but because the increase is directly correlated with an increase in clients and the hospital's activity.

Accordingly, the first recommendation results (1) there should be a goal-oriented remuneration, where more activity of excellence is encouraged. Note that, this activity must take into consideration not only the quantity but also the quality of performance. The second recommendation arises in alignment with the present dissertation (2) investments in innovative methodologies to measure the hospital's efficiency should be a priority to optimize future decisions based on previous accomplishments.

6.2. Limitations & future work

As far as limitations are concerned, the major one consists in the heterogeneity regarding variables quantity for the different divisions. In other words, the different quantities of internal variables may have resulted in the amplification of dissimilarities among methodologies. Additionally, finding the appropriated variables is normally difficult, as point by (Ozcan, 2008). However, the inclusion of environmental factors, detailed in Subsection 5.4, such as the legionella outbreak in 2018, provided a major insightful into understanding the causes of inefficiency.

As future research, it would be graceful to overcome the limitations previously presented by homogenizing the number of variables used in each division under investigation. Through analysis of the efficiency scores obtained in the next researches, it would facilitate the comprehension of the differences between the application of standard and network DEA. In other words, it is fundamental to study of the impact of specific connections in the determination of global efficiency.

Hereafter, in an academia standpoint, it would be interesting to continue the comparison of efficiency methodologies. On one hand, deepen the investigation of the advantages and disadvantages of the incorporation of internal connections in efficiency analysis. For example, by applying different models than the one developed at the present dissertation to compare standard and network DEA. On the other hand, it would be enlightening to apply the same technique to the public healthcare sector and compare the results obtained. As well as ex-

tending the network DEA to a dynamic setting and meta- or partial frontiers capable of adjusting when facing environmental variations (Lobo et al., 2016; Yu and Chen, 2020).

To conclude it would be an exquisite honour to witness the introduction of the hospital's holistic efficiency analysis. The incorporation of internal connections would benefit the decision-making process, as well as comprehension of the interconnectivity of business and social sectors to overcome real-world challenges.

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7. Annexes

Table 1: Inputs and outputs, indicators, and respective descriptions for standard DEA analysis.

Type	Indicator	Description
Input (m=7)	FTE health auxiliaries, x_{1j}	Reports the full-time equivalent that indicates the health auxiliaries' workload.
	FTE nurses, x_{2j}	Reports the full-time equivalent that indicates the nurses' workload.
	FTE Doctor with a contract, x_{3j}	Reports the full-time equivalent that indicates the fixed doctors' workload.
	FTE health technicians, x_{4j}	Reports the full-time equivalent that indicates the health technicians' workload.
	FTE Doctor without a contract, x_{5j}	Reports the full-time equivalent that indicates the non-fixed doctors' workload.
	Number of beds, x_{6j}	Details the total DMU_j 's number of beds.
	Operating Costs, x_{7j}	Corresponds to the total expenses (in €) of a DMU .
Output (s=3)	Inpatients, y_{1j}	Matches the number of patients leaving the inpatient service.
	Total number of clients, y_{2j}	Outlines the absolute number of clients conducted by the DMU_j .
	Hospitalizations, y_{3j}	Appraises the absolute number of medical hospitalizations.

Table 2: Inputs, intermediate variables, and outputs, indicators, respective descriptions, and correspondent division for network DEA analysis.

Type	Indicator	Description	Division
Inputs	FTE health auxiliaries, x_{1j}	Reports the full-time equivalent that indicates the health auxiliaries' workload.	1-3
	FTE nurses, x_{2j}	Reports the full-time equivalent that indicates the nurses' workload.	1-3
	FTE Doctor with a contract, x_{3j}	Reports the full-time equivalent that indicates the fixed doctors' workload.	1-3
	FTE health technicians, x_{4j}	Reports the full-time equivalent that indicates the health technicians' workload.	1 and 2
	FTE Doctor without a contract, x_{5j}	Reports the full-time equivalent that indicates the non-fixed doctors' workload.	1-3
	Number of beds, x_{6j}	Details the total DMU_j 's number of beds.	2 and 3
	Operating Costs, x_{7j}	Corresponds to the total expenses (in €) of a DMU .	1-3
Intermediate variables	Number of episodes, z_{1j}	Outlines the absolute number of hospitalization episodes that result from consultation or permanent assistance	1 → 2 and 3 → 2
	Number of patients, z_{2j}	Corresponds to the absolute number of patients who move from permanent assistance to hospitalization division	3 → 2
Outputs	Inpatients, y_{1j}	Matches the number of patients leaving the inpatient service.	1 and 3
	Total number of clients, y_{2j}	Outlines the absolute number of clients conducted by the DMU_j .	1-3
	Hospitalizations, y_{3j}	Appraises the absolute number of medical hospitalizations.	2

Table 3: Number of efficient units for each division for both methodologies, standard DEA and network DEA

Division	# efficient DMUs using standard DEA	# efficient DMUs using network DEA	Error
Division 1	48	52	4 units
Division 2	46	73	27 units
Division 3	20	32	12 units

Table 4: The number of globally efficient units for both methodologies, standard DEA, and network DEA.

# efficient DMUs using standard DEA	# efficient DMUs using network DEA	Difference
63	20	43 units